

The planets, comets and binary stars

An observer looking up at the night sky from the surface of the Earth sees an unchanging pattern of stars revolving slowly about the pole as the Earth spins on its axis. So great are the distances to the stars that the changing position of the Earth as it travels along its orbit around the Sun causes hardly any movement in the pattern, even in the course of six months. There are a few objects, however, which do appear to move a great deal with respect to this fixed background of stars. The objects are members of our Solar System, the planets, the asteroids and the comets. Eight major planets have been identified so far which, in order of increasing distance from the Sun, are Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, and Neptune. (Pluto was originally classified as a planet, but is now considered the largest member of another grouping of objects called the Kuiper belt.) These, together with other members of the Solar System, are all bound by the gravitational field of the Sun so that instead of moving off into space in different directions they are constrained to follow elliptical orbits about it. Their apparent motions in the sky are complicated because they are relatively close to us so that the position of the Earth in its own orbit needs to be taken into account. The next few sections contain methods for calculating the positions, angular sizes, distances, phases and brightnesses of the major planets. There are also sections describing how to calculate the orbit of a comet and the orbit of a binary star.

53 The planetary orbits

Each planet in our Solar System describes an elliptical orbit about the Sun with the Sun at a focus of the ellipse. We discovered how to calculate the Sun–Earth orbit in Sections 44 to 47. This was a particularly simple case since the plane of the orbit defined the plane of the ecliptic; the ecliptic latitude was therefore always zero and the fundamental direction, the first point of Aries, was in the orbital plane. The other planets, however, do not move in the plane of the ecliptic but describe orbits inclined at small angles to it. Figure 63 shows the situation.

The Sun, S , is at the centre of the diagram and you are to imagine that you are looking at the path of a planet around the Sun from a great distance. The orbit of the planet is the small shaded ellipse N_1AP . The perihelion is marked A and the planet's present position is marked P . That part of the orbit which lies above the ecliptic is shown with solid lines, while that lying below it is shown with dashed lines. The large sphere is centred on the Sun and the plane of the planet's orbit is projected to cut the sphere along the circle $N_1A'P'N_2$. Here A' is the projection of A onto the sphere, P' the projection of P and so forth. Also shown in the diagram is the plane of the ecliptic $\Upsilon N_1'N_2'$, which contains the direction of Υ , the first point of Aries.

The planet moves along its orbit in the direction of the arrow. The point N_1 where it rises out of the plane of the ecliptic is called the **ascending node**. N_2 , the point where it descends below the plane of the ecliptic, is called the **descending node**. Angles in the orbital plane are measured from the ascending node while longitudes are reckoned from the direction Υ which is not in the orbital plane. Thus the perihelion is at an angle ω to the node (the 'argument' of the perihelion) and the present position of the planet is at an angle $\omega + v$. The corresponding longitudes are $\omega + \Omega$ and $\omega + v + \Omega$, where Ω is the longitude of the ascending node. Note that longitudes are the sum of two angles in different planes.

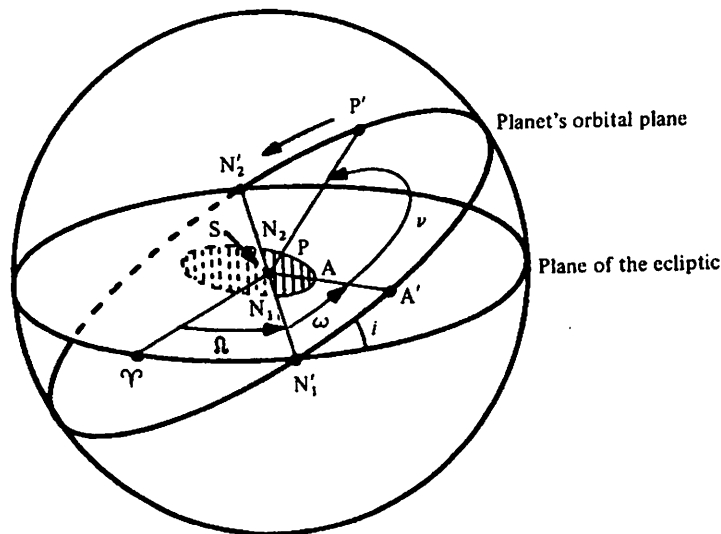


Figure 63. Defining the orbit of a planet.

54 Calculating the coordinates of a planet

Our calculation will proceed in three steps. The first is to calculate the position of the planet in its own orbital plane exactly as we did for the Sun–Earth orbit in Section 46. In the second step we will project the planet's calculated position onto the plane of the ecliptic and hence find its ecliptic longitude and latitude referred to the Sun (**heliocentric coordinates**). The third step will involve transforming from the Sun to the Earth to find the ecliptic coordinates referred to the Earth, from which we can find the right ascension and declination by the method given in Section 27.

As before, we choose our starting point, the epoch, as 2010.0. Having calculated the number of days, D , since the epoch, we find the mean anomaly, M , by the formula

$$M = \frac{360}{365.242\,191} \times \frac{D}{T_p} + \varepsilon - \varpi \text{ degrees,}$$

where T_p is the orbital period of the planet in tropical years, ε is the mean longitude of the planet at the epoch, and ϖ is the longitude of the perihelion. These constants are listed for the planets in our Solar System in Table 8. This table is extracted from a list of **osculating elements** published on the web by the US Naval Observatory (see page 209).

Being osculating elements, they change with time and are valid only over a relatively short period. We can use the values in Table 8 for low-precision calculations, but should use the more-precise spreadsheet of Section 56 for extrapolations into the past or future of more than a few tens of years from 2010, or where higher accuracy is needed.

The mean anomaly refers to the motion of a fictitious planet, P_1 , moving in a circle at constant speed with the same orbital period as the real planet (see Figure 64). We really want to know the value of the true anomaly, v , which is the angle the real planet actually makes with the line joining the Sun to the perihelion. We can find v from M using the equation of the centre:

$$v = M + \frac{360}{\pi} e \sin M \text{ degrees,}$$

where e is the eccentricity of the orbit (Table 8) and $\pi = 3.141\,592\,7$. Once again, this formula is an approximation that is good enough for most purposes; if you wish to make more precise calculations you can find the value of v by solving Kepler's equation via the method of Section 47.

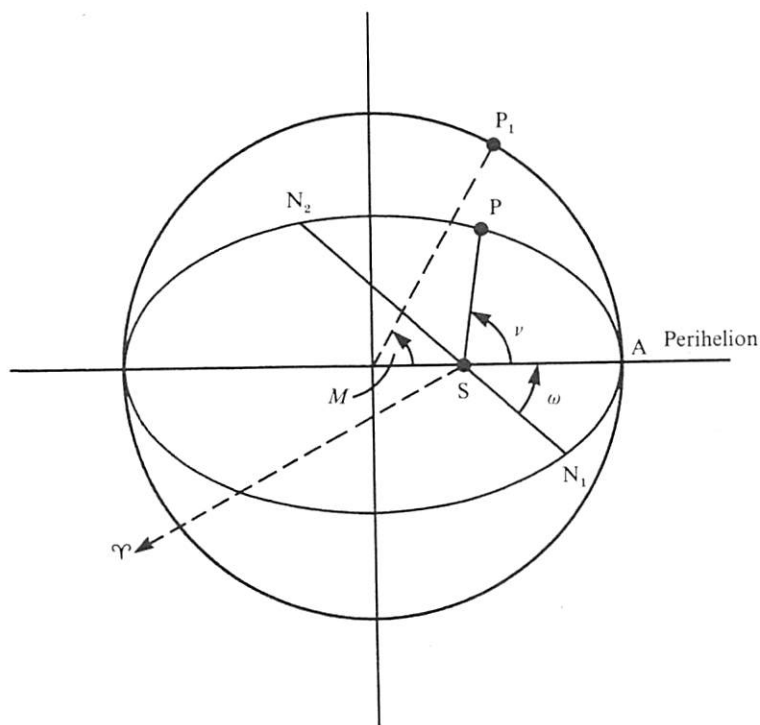


Figure 64. Mean and true anomalies.

	T_p (tropical years)	ε (degrees)	ϖ (degrees)	e	a (AU)	i (degrees)	Ω (degrees)	θ_0 (arcsec)	V_0
Mercury	0.24085	75.5671	77.612	0.205627	0.387098	7.0051	48.449	6.74	-0.42
Venus	0.615207	272.30044	131.54	0.006812	0.723329	3.3947	76.769	16.92	-4.40
Earth	0.999996	99.556772	103.2055	0.016671	0.999985				
Mars	1.880765	109.09646	336.217	0.093348	1.523689	1.8497	49.632	9.36	-1.52
Jupiter	11.857911	337.917132	14.6633	0.048907	5.20278	1.3035	100.595	196.74	-9.40
Saturn	29.310579	172.398316	89.567	0.053853	9.51134	2.4873	113.752	165.60	-8.88
Uranus	84.039492	271.063148	172.884833	0.046321	19.21814	0.773059	73.926961	65.80	-7.19
Neptune	165.84539	326.895127	23.07	0.010483	30.1985	1.7673	131.879	62.20	-6.87

1 AU = 149.6×10^6 km.

T_p : period of orbit; ε : longitude at the epoch; ϖ : longitude of the perihelion; e : eccentricity of the orbit; a : semi-major axis of the orbit; i : orbital inclination; Ω : longitude of the ascending node; θ_0 : angular diameter at 1 AU; V_0 : visual magnitude at 1 AU.

Table 8. Elements of the planetary orbits at epoch 2010.0.

The next step is to calculate the heliocentric longitude, l , and this is simply given by

$$l = v + \varpi,$$

or

$$l = \left(\frac{360}{365.242\,191} \times \frac{D}{T_p} \right) + \frac{360}{\pi} e \sin \left(\frac{360}{365.242\,191} \times \frac{D}{T_p} + \varepsilon - \varpi \right) + \varepsilon \text{ degrees.}$$

We also need the length of the radius vector, r , calculated from

$$r = \frac{a(1 - e^2)}{1 + e \cos v},$$

where a is the semi-major axis of the orbit (Table 8).

The above calculations which you have made for the planet have to be repeated for the Earth as well. We shall denote the values derived for the planet by small letters and use capital letters for the Earth's values. Thus, we arrive at the figures for l and r for the planet and L and R for the Earth. In addition, we need the heliocentric latitude of the planet:

$$\psi = \sin^{-1} \{ \sin(l - \varpi) \sin i \},$$

where i is the inclination of the orbit and ϖ is the longitude of the ascending node (Table 8). The heliocentric latitude of the Earth is, of course, zero.

Now we need to project our calculations for the planet onto the plane of the ecliptic to find the projected heliocentric longitude, l' , and the projected radius vector, r' . These are given by the formulas

$$l' = \tan^{-1} \{ \tan(l - \varpi) \cos i \} + \varpi,$$

$$r' = r \cos \psi.$$

The final step in the process is to refer the calculations to the Earth to find the geocentric ecliptic latitude, β , and longitude, λ , of the planet. Figure 65(a) describes the situation for an **outer planet**, whose orbit lies outside that of the Earth (i.e. Mars, Jupiter, Saturn, Uranus and Neptune), and Figure 65(b) is for an **inner planet** (the inner planets are Mercury and Venus). The plane of the paper represents the plane of the ecliptic. S is the Sun, E is the Earth and P₁ is the position of the planet projected onto the ecliptic. The first point of Aries is taken to be at a distance from the Solar System so large that the directions E Υ and S Υ are parallel. Then by application of a little simple geometry we have for the outer planets

$$\lambda = \tan^{-1} \left\{ \frac{R \sin(l' - L)}{r' - R \cos(l' - L)} \right\} + l' \text{ degrees,}$$

and for the inner planets

$$\lambda = 180 + L + \tan^{-1} \left\{ \frac{r' \sin(L - l')}{R - r' \cos(L - l')} \right\} \text{ degrees,}$$

Figure 66 gives the diagram for calculating the latitude. Again, using simple geometry we find

$$\beta = \tan^{-1} \left\{ \frac{r' \tan \psi \sin(\lambda - l')}{R \sin(l' - L)} \right\} \text{ degrees,}$$

true for inner and outer planets alike.

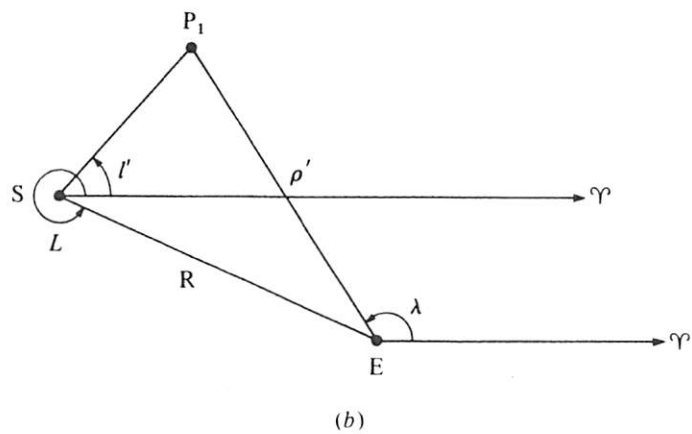
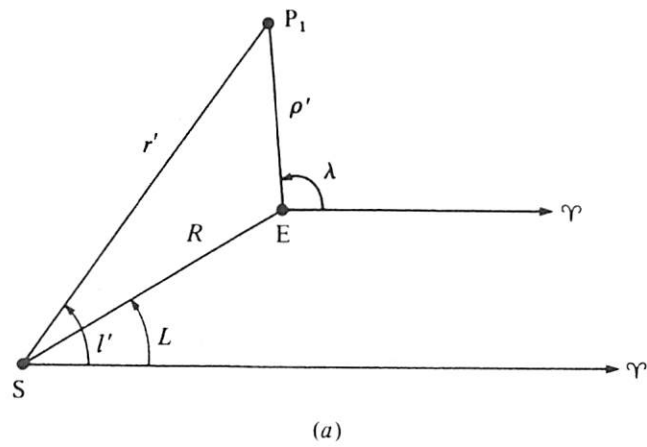


Figure 65. Ecliptic geometry: (a) outer planet, (b) inner planet.

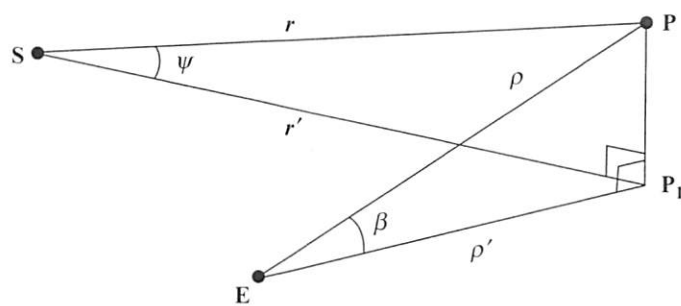


Figure 66. Projecting onto the ecliptic.

Let us consolidate these rather lengthy calculations with two examples, one for an outer planet, Jupiter, and the other for an inner planet, Mercury. For each of these two planets we shall calculate its right ascension and declination on 22 November 2003. For Jupiter (outer planet):

Method	Example
1. Find the number of days since 2010 January 0.0 (§3). The total is D .	22 November = 304 + 22 = 326 -2 557 D = -2231 days
<i>For the planet, Jupiter:</i>	
2. Calculate $N_p = \frac{360}{365.242191} \times \frac{D}{T_p}$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	N_p = 174.555932 degrees
3. Find $M_p = N_p + \varepsilon - \varpi$.	M_p = 497.809764 degrees
4. Calculate $v_p = M_p + \frac{360}{\pi} e \sin M_p$ degrees. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	v_p = 141.573600 degrees
5. Find $l_p = v_p + \varpi$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	l_p = 156.236900 degrees
6. Calculate $r = \frac{a(1-e^2)}{1+e \cos v_p}$.	r = 5.397121 AU
<i>Now do the calculations for the Earth:</i>	
7. Calculate $N_E = \frac{360}{365.242191} \times \frac{D}{T_E}$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	N_E = 321.011952 degrees
8. Find $M_E = N_E + \varepsilon_E - \varpi_E$.	M_E = 317.363223 degrees
9. Calculate $v_E = M_E + \frac{360}{\pi} e_E \sin M_E$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	v_E = 316.069248 degrees
10. Find $L = v_E + \varpi_E$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	L = 59.274748 degrees
11. Calculate $R = \frac{a_E(1-e_E^2)}{1+e_E \cos v_E}$.	R = 0.987847 AU

Method (continued)	Example
<i>More calculations for the planet:</i>	
12. Calculate $\psi = \sin^{-1} \{ \sin (l_p - \delta_\odot) \sin i \}$.	$\psi = 1.076\,044$ degrees
13. Find $y = \sin (l_p - \delta_\odot) \cos i$.	$y = 0.825\,313$
14. Find $x = \cos (l_p - \delta_\odot)$.	$x = 0.564\,363$
15. Find $\tan^{-1} \left(\frac{y}{x} \right)$ and remove the ambiguity by referring to Figure 29, adding or subtracting 180 degrees as indicated by the signs of x and y so that the result lies in the correct quadrant.	$\tan^{-1} \left(\frac{y}{x} \right) = 55.634\,991$ y and x both positive so the result is in the right quadrant
16. Add δ_\odot to get l' (check: l' should be nearly equal to l_p).	$l' = 156.229\,991$ degrees
17. Find $r' = r \cos \psi$.	$r' = 5.396\,170$ AU
<i>Combine the calculations:</i>	
18. Calculate $\lambda = \tan^{-1} \left\{ \frac{R \sin(l' - L)}{r' - R \cos(l' - L)} \right\} + l'$. Bring the result into the range 0 to 360 by adding or subtracting 360. This is the planet's geocentric ecliptic longitude.	$\lambda = 166.310\,510$ degrees
19. Find $\beta = \tan^{-1} \left\{ \frac{r' \tan \psi \sin(\lambda - l')}{R \sin(l' - L)} \right\}$. This is the planet's geocentric ecliptic latitude.	$\beta = 1.036\,466$ degrees
20. Finally, convert λ and β to right ascension and declination (§27).	$\alpha = 11^{\text{h}}\,11^{\text{m}}\,14^{\text{s}}$ $\delta = 6^\circ\,21'\,25''$

The *Astronomical Almanac* gives the apparent coordinates of Jupiter for this day as $\alpha = 11\text{h } 10\text{m } 30\text{s}$ and $\delta = 6^\circ 25' 56''$. The error due to our approximation in counting only the first term of the equation of the centre could be reduced by solving Kepler's equation using the method in Section 47. We will see how to make more exact calculations in Section 56. For Mercury (inner planet):

Method	Example
1. We proceed exactly as we did in the previous example, calculating l_p , v_p , r , ψ , l' and r' for Mercury, and L , v_E and R for the Earth.	$l_p = 288.012\,253$ degrees $v_p = 210.400\,253$ degrees $r = 0.450\,657$ AU $\psi = -6.035\,842$ degrees $l' = 287.824\,406$ degrees $r' = 0.448\,159$ degrees $L = 59.274\,748$ degrees $v_E = 316.069\,248$ degrees $R = 0.987\,847$ AU
2. Now calculate $\lambda = 180 + L + \tan^{-1} \left\{ \frac{r' \sin(L-l')}{R-r' \cos(L-l')} \right\}$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	$\lambda = 253.929\,758$ degrees
3. Find $\beta = \tan^{-1} \left\{ \frac{r' \tan \psi \sin(\lambda-l')}{R \sin(l'-L)} \right\}$.	$\beta = -2.044\,057$ degrees
4. Finally, convert λ and β to right ascension and declination (§27).	$\alpha = 16\text{h } 49\text{m } 12\text{s}$ $\delta = -24^\circ 30' 09''$

The *Astronomical Almanac* gives the apparent coordinates of planet Mercury as $\alpha = 16\text{h } 52\text{m } 02\text{s}$ and $\delta = -24^\circ 38' 41''$. We should generally expect an error in α of a few minutes at most and in δ of a quarter of a degree, but the errors may be more for Mercury, for which $e = 0.2$. The inaccuracies arise because we have used only the first term in the equation of the centre, we have not allowed for the light travel time, and because of the slight perturbations to the orbits from other planets in the Solar System (see Section 56). We could reduce the error from the first cause by using the longer method of Section 47; see Figure 68 for a graph of the error incurred by the shorter method.

The spreadsheet for this calculation, called PlanetPos1, Figure 67 (three panels), uses a technique which we have not met previously in this book. The parameters of the orbits of the planets are presented in a table in a separate spreadsheet called Planet data (third panel). This table reproduces most of the data in Table 8, but note that the order of the planets is now alphabetical. The planet name is in column A and is used as the key to the corresponding row of data contained in columns C to I inclusive. If we number the columns in the table, then column 1 contains the planet name, 3 contains the orbital period, 4 the longitude at the epoch and so on. We can obtain any element of the data using the spreadsheet function VLOOKUP (e.g. row 17 of the first panel of Figure 67). This takes four arguments, which are the planet name (upper or lower case, or a mixture), the range of the table from top-left-hand corner to bottom-right-hand corner (e.g. 'Planet data'!A3:I10), the column number containing the required element (e.g. 3 for the orbital period, 4 for the longitude at the epoch), and a switch which is set to TRUE to find either an approximate or an exact match with the planet name in column 1, or FALSE if an exact match is required (as here). Using this formula makes it easy to change the orbital parameters without affecting the main spreadsheet calculation. Simply fill in the Planet data table with new numbers, and the spreadsheet will use those instead. The formulas contained in cells G3 to G8 are shown in cells G10 to G15 to save space.

	A	B	C	D	E	F	G	H	I	J	K	L
1	The positions of the planets (approximate method)											
2												
3	Input	local civil time (hour)	0	Output	Jupiter RA (hour)	11	=DHHour(C61)					
4		local civil time (min)	0		Jupiter RA (min)	11	=DHMin(C61)					
5		local civil time (sec)	0		Jupiter RA (sec)	13.8	=DHSec(C61)					
6		daylight saving (hours)	0		Jupiter dec (deg)	6	=DDDeg(C62)					
7		zone correction (hours)	0		Jupiter dec (min)	21	=DDMin(C62)					
8		local date (day)	22		Jupiter dec (sec)	25.1	=DDSec(C62)					
9		local date (month)	11									
10		local date (year)	2003									
11		planet name	Jupiter				=CONCATENATE(SC\$11," RA (hour)")					
12							=CONCATENATE(SC\$11," RA (min)")					
13							=CONCATENATE(SC\$11," RA (sec)")					
14							=CONCATENATE(SC\$11," dec (deg)")					
15							=CONCATENATE(SC\$11," dec (min)")					
16							=CONCATENATE(SC\$11," dec (sec)")					
17	1	planet Tp from table	11.857911	=VLOOKUP(C11,'Planet data'!A3:I10,3,FALSE)								
18	2	planet long from table	337.917132	=VLOOKUP(C11,'Planet data'!A3:I10,4,FALSE)								
19	3	planet peri from table	14.6633	=VLOOKUP(C11,'Planet data'!A3:I10,5,FALSE)								
20	4	planet ecc from table	0.048907	=VLOOKUP(C11,'Planet data'!A3:I10,6,FALSE)								
21	5	planet axis from table	5.20278	=VLOOKUP(C11,'Planet data'!A3:I10,7,FALSE)								
22	6	planet incl from table	1.3035	=VLOOKUP(C11,'Planet data'!A3:I10,8,FALSE)								
23	7	planet node from table	100.595	=VLOOKUP(C11,'Planet data'!A3:I10,9,FALSE)								
24	8	Gdate (day)	22	=LctGDay(C3,C4,C5,C6,C7,C8,C9,C10)								
25	9	Gdate (month)	11	=LctGMonth(C3,C4,C5,C6,C7,C8,C9,C10)								
26	10	Gdate (year)	2003	=LctGYear(C3,C4,C5,C6,C7,C8,C9,C10)								
27	11	UT (hours)	0	=LctUT(C3,C4,C5,C6,C7,C8,C9,C10)								
28	12	D (days)	-2231	=CDJD(C24+(C27/24),C25,C26)-CDJD(0,1,2010)								
29	13	Np (deg)	-185.4440679	=360*C28/(365.242191*C17)								
30	14	Np (deg)	174.5559321	=C29-360*INT(C29/360)								
31	15	Mp (deg)	497.8097641	=C30+C18-C19								
32	16	Lp (deg)	516.2369	=C30+(360*C20*SIN(RADIANS(C31))/PI())/C18								

Figure 67. Finding the position of a planet by an approximate method; panels one and two show the main spreadsheet, and the third shows the data table (continued on the next page). The formulas contained in cells G3 to G8 are shown in cells G10 to G15 to save space.

	A	B	C	D	E	F	G	H	I	J	K	L
33	17	L p (deg)	156.2369	=C32-360*INT(C32/360)								
34	18	planet true anomaly (deg)	141.5736	=C33-C19								
35	19	r (AU)	5.397121314	=C21*(1-(C20)^2)/(1+C20*COS(RADIANS(C34)))								
36	20	Earth Tp from table	0.999996	=VLOOKUP("Earth",Planet data!A3:110.3,FALSE)								
37	21	Earth long from table	99.556772	=VLOOKUP("Earth",Planet data!A3:110.4,FALSE)								
38	22	Earth peri from table	103.2055	=VLOOKUP("Earth",Planet data!A3:110.5,FALSE)								
39	23	Earth ecc from table	0.016671	=VLOOKUP("Earth",Planet data!A3:110.6,FALSE)								
40	24	Earth axis from table	0.999985	=VLOOKUP("Earth",Planet data!A3:110.7,FALSE)								
41	25	Ne (deg)	-2198.988048	=360*C28/(365.242191*C36)								
42	26	Ne (deg)	321.0119519	=C41-360*INT(C41/360)								
43	27	Me (deg)	317.3632239	=C42+C37-C38								
44	28	Le (deg)	419.2747476	=C42+C37+360*C39*SIN(RADIANS(C43))/PI()								
45	29	Le (deg)	59.27474763	=C44-360*INT(C44/360)								
46	30	Earth true anomaly (deg)	-43.93075237	=C45-C38								
47	31	R (AU)	0.987846892	=C40*(1-(C39)^2)/(1+C39*COS(RADIANS(C46)))								
48	32	(L p-Node) (rad)	0.971134357	=RADIANS(C33-C23)								
49	33	psi (rad)	0.018780513	=ASIN(SIN(C48)*SIN(RADIANS(C22)))								
50	34	y	0.825312806	=SIN(C48)*COS(RADIANS(C22))								
51	35	x	0.564363453	=COS(C48)								
52	36	ld (deg)	156.229991	=DEGREES(ATAN2(C51,C50))+C23								
53	37	rd (AU)	5.396169539	=C35*COS(C49)								
54	38	Le-Ld (rad)	-1.692188223	=RADIANS(C45-C52)								
55	39	ATAN2 type 1	-1.273466183	=ATAN2(C47-C53*COS(C54),C53*SIN(C54))								
56	40	ATAN2 type 2	0.175938247	=ATAN2(C53-C47*COS(C54),C47*SIN(-C54))								
57	41	A (rad)	0.175938247	=IF(C53<1,C55,C56)								
58	42	lamda (deg)	166.31051	=IF(C53<1,180+C45+DEGREES(C57),DEGREES(C57)+C52)								
59	43	lamda (deg)	166.31051	=C58-360*INT(C58/360)								
60	44	beta (deg)	1.036465596	=DEGREES(ATAN(C53*TAN(C49)*SIN(RADIANS(C59-C52))/(C47*SIN(-C54))))								
61	45	RA (hours)	11.1871665	=DDDH(ECRA(C59,0,0,C60,0,0,C24,C25,C26))								
62	46	dec (deg)	6.356972099	=ECDec(C59,0,0,C60,0,0,C24,C25,C26)								

	A	B	C	D	E	F	G	H	I
1	Planet name		Tp	Long	Peri	Ecc	Axis	Incl	Node
2									
3	EARTH		0.999996	99.556772	103.2055	0.01667	0.99999		
4	JUPITER		11.85791	337.91713	14.6633	0.04891	5.20278	1.3035	100.595
5	MARS		1.880765	109.09646	336.217	0.09335	1.52369	1.8497	49.632
6	MERCURY		0.24085	75.5671	77.612	0.20563	0.3871	7.0051	48.449
7	NEPTUNE		165.8454	326.89513	23.07	0.01048	30.1985	1.7673	131.879
8	SATURN		29.31058	172.39832	89.567	0.05385	9.51134	2.4873	113.752
9	URANUS		84.03949	271.06315	172.88483	0.04632	19.2181	0.77306	73.926961
10	VENUS		0.615207	272.30044	131.54	0.00681	0.72333	3.3947	76.769

Figure 67. (Continued.)

55 Finding the approximate positions of the planets

The method of finding the equatorial coordinates of the planets given in the previous section is quite accurate but involves lengthy calculations. An amateur astronomer often only wants to know the approximate position of a planet so that he or she knows where to look for it in the sky, and does not want to have to spend 20 minutes beforehand submerged in a sea of figures obtaining the information. In that case it is usually sufficient to assume that the planets describe circular orbits about the Sun which all lie in the plane of the ecliptic. This leads to considerable simplifications in the calculations.

The heliocentric longitude, l , does not have to be corrected by the equation of the centre so that we may write

$$l = \frac{360}{365.242\,191} \times \frac{D}{T_p} + \varepsilon \text{ degrees.}$$

We repeat this calculation for the Earth as before (giving L). Since the orbits are assumed to be circular with the Sun at the centre, the radius vector is constant. Hence

$$r = a.$$

The heliocentric (and therefore the geocentric) latitude of the planet is zero since we have assumed that the orbit lies in the ecliptic plane. Our final calculation is therefore

$$\lambda = \tan^{-1} \left\{ \frac{\sin(l-L)}{a - \cos(l-L)} \right\} + l, \quad (\text{Outer})$$

for the outer planets and

$$\lambda = 180 + L + \tan^{-1} \left\{ \frac{a \sin(L-l)}{1 - a \cos(L-l)} \right\}, \quad (\text{Inner})$$

for the inner planets, since we have assumed that $R = 1$ (the Earth's orbital radius is taken to be unity). This is the geocentric longitude of the planet from which the right ascension and declination can be found using the formulas of Section 27 (remember $\beta = 0$):

$$\alpha = \tan^{-1} \{ \tan \lambda \cos \varepsilon \},$$

$$\delta = \sin^{-1} \{ \sin \lambda \sin \varepsilon \},$$

where ε here is the obliquity of the ecliptic (about 23.5 degrees; see Section 27). In some cases it may even be possible to ignore the fact that the plane of the ecliptic is inclined at an angle to the plane of the equator and to write

$$\alpha = \lambda.$$

As an example, we will calculate again the coordinates of Jupiter on 22 November 2003 using this approximate method.

Method	Example
1. Find the number of days since 2010 January 0.0 (§3). The total is D .	22 November = 304 + 22 = 326 – 2557 D = –2 231 days
2. Calculate $l = \left(\frac{360}{365.242\,191} \times \frac{D}{T_p} \right) + \varepsilon$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	l = 152.47 degrees
3. Repeat step 2 for the Earth to find L .	L = 60.57 degrees
4. Calculate $\lambda = \tan^{-1} \left\{ \frac{\sin(l-L)}{a - \cos(l-L)} \right\} + l$. Bring the result into the range 0–360 by adding or subtracting 360. This is the planet's geocentric ecliptic longitude.	λ = 163.28 degrees
5. Finally, convert λ and β ($= 0$) to right ascension and declination (§27).	α = 10h 58m δ = 6° 34'

56 Perturbations in a planet's orbit

Throughout the calculations to find the coordinates of a planet (Section 54), we assumed that its motion was controlled entirely by the gravitational field of the Sun so that the influences of other members of the Solar System were negligible. This is true to quite a high accuracy, but for more precision we need to take account of these **perturbations**, especially for the orbits of Jupiter and Saturn where the effects can be as large as 1 degree in longitude. The usual method of doing so is to apply a series of correction terms to the quantities calculated in Section 54. We have to make similar adjustments for the Moon in Section 65. Here, we shall consider only the most important terms in the orbits of Jupiter and Saturn where the corrections amount to more than about 0.04 degrees in longitude.

We must first calculate the time, T , in Julian centuries since the epoch 1900 January 0.5. This is given by

$$T = \frac{\text{JD} - 2415\,020.0}{36\,525},$$

where JD is the Julian date (Section 4).[†] Then we calculate the quantities:

$$A = \frac{T}{5} + 0.1,$$

$$P = 237.47555 + 3\,034.906\,1T \text{ degrees},$$

$$Q = 265.91650 + 1\,222.1139T \text{ degrees},$$

$$V = 5Q - 2P, \text{ and}$$

$$B = Q - P.$$

[†]Note that this definition of T differs from that used in other parts of the book.

The principal terms for Jupiter and Saturn are then:

$$\text{Jupiter: } \Delta l = (0.3314 - 0.0103A) \sin V - 0.0644A \cos V \text{ degrees.}$$

$$\text{Saturn: } \Delta l = (0.1609A - 0.0105) \cos V + (0.0182A - 0.8142) \sin V - 0.1488 \sin B \\ - 0.0408 \sin 2B + 0.0856 \sin B \cos Q + 0.0813 \cos B \sin Q \text{ degrees.}$$

The value of Δl must be added to the mean longitude l before proceeding with the calculation of Section 54.

Let us now recalculate the position of Jupiter on 22 November 2003, solving Kepler's equation properly (Section 47) and allowing for these principal terms of perturbation.

Method	Example
1. Calculate the Julian date (§4).	JD = 2 452 965.5 days
2. Find $T = \frac{\text{JD} - 2415020.0}{36525}$.	$T = 1.038891$ centuries
3. Find $A = \left(\frac{T}{5}\right) + 0.1$.	$A = 0.307778$ centuries
4. Calculate $P = 237^\circ.47555 + 3034^\circ.9061 T$.	$P = 3390.412183$ degrees
5. Find $Q = 265^\circ.91650 + 1222^\circ.1139 T$.	$Q = 1535.559632$ degrees
6. Find $V = 5Q - 2P$.	$V = 896.973792$ degrees
7. Calculate $\Delta l = (0.3314 - 0.0103A) \sin V - 0.0644A \cos V$.	$\Delta l = 0.037121$ degrees
8. Now proceed as in the example of §54 to find M_p .	$M_p = 497.809764$ degrees
9. Use the method of §47 to find v_p .	$E_p = 2.436915$ radians
	$v_p = 141.407886$ degrees
10. Find $l_p = v_p + \varpi$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	$l_p = 156.071186$ degrees
11. Add Δl to get a better estimate of l_p .	$l_p = 156.108307$ degrees
12. Calculate L , v_E and R for the Earth also using the method of §47.	$E_E = 5.527600$ radians
	$v_E = 316.049185$ degrees
	$L = 59.254685$ degrees
	$R = 0.987847$ AU
13. Now proceed with the calculations of §54 to find α and δ .	$r = 5.396627$ AU
	$\psi = 0.018751$ degrees
	$l' = 156.101386$ degrees
	$r' = 5.395678$ AU
	$\lambda = 166.188415$ degrees
	$\beta = 1.035198$ degrees
	$\alpha = 11^h 10^m 47^s$
	$\delta = 6^\circ 24' 12''$

The error incurred in considering only the first term of the equation of the centre is plotted as a function of the mean anomaly, M , in Figure 68 for two values of the eccentricity, e .

The spreadsheet for finding the positions of the planets by a more precise method is called PlanetPos2 and is shown in Figure 69. We have used a full numerical model of the orbits of the planets in the two spreadsheet functions PlanetLong and PlanetLat which return, respectively, the named planet's ecliptic longitude and latitude in degrees. These functions appear in rows 20 and 21. They each take nine arguments, namely the local civil time expressed as hours, minutes, and seconds, the daylight saving correction and zone time offset in hours, the local calendar date expressed as day, month, year, and the full name of the planet as a string in upper or lower case. The example shown is for Jupiter on the 22 November 2003, and you can see that this method has provided the position correct within about 1 second in right ascension, and within a few arcseconds in declination. The formulas in cells G3 to G8 are shown in cells H10 to H15 to save space.

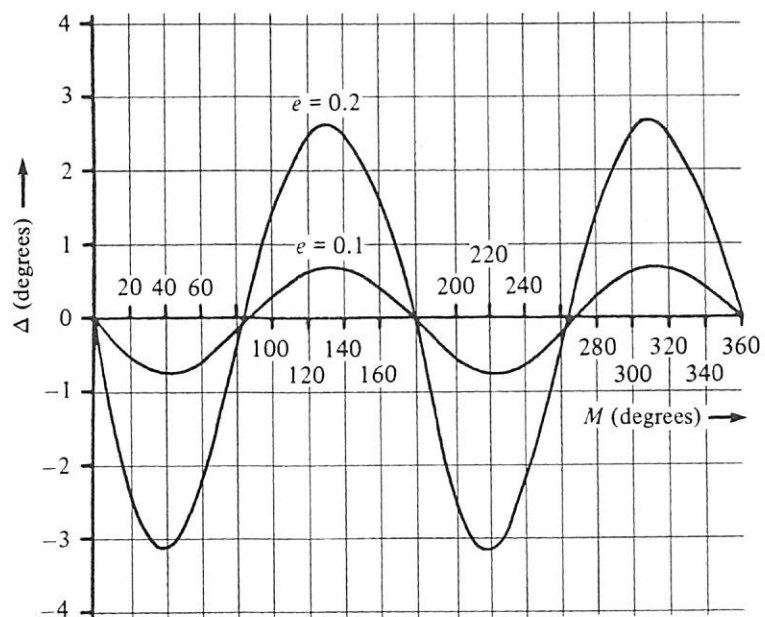


Figure 68. The error, Δ , incurred by taking $v = M + (360/\pi)e \sin M$ as an approximation to elliptical motion. The true anomaly should be $v - \Delta$.

	A	B	C	D	E	F	G	H	I	J	K
	The positions of the planets (more precise method)										
1											
2											
3	Input	local civil time (hour)	0	Output	Jupiter RA (hour)		11 =DHHour(C22)				
4		local civil time (min)	0		Jupiter RA (min)		10 =DHMin(C22)				
5		local civil time (sec)	0		Jupiter RA (sec)		30.99 =DDSec(C22)				
6		daylight saving (hours)	0		Jupiter dec (deg)		6 =DDDeg(C23)				
7		zone correction (hours)	0		Jupiter dec (min)		25 =DDMin(C23)				
8		local date (day)	22		Jupiter dec (sec)		49.46 =DDSec(C23)				
9		local date (month)	11								
10		local date (year)	2003								
11		planet name	Jupiter				=CONCATENATE(\$C\$11," RA (hour)")				
12							=CONCATENATE(\$C\$11," RA (min)")				
13							=CONCATENATE(\$C\$11," RA (sec)")				
14							=CONCATENATE(\$C\$11," dec (deg)")				
15							=CONCATENATE(\$C\$11," dec (min)")				
16							=CONCATENATE(\$C\$11," dec (sec)")				
17	1	Greenwich date (day)	22 =LctGDay(C3,C4,C5,C6,C7,C8,C9,C10)								
18	2	Greenwich date (month)	11 =LctGMonth(C3,C4,C5,C6,C7,C8,C9,C10)								
19	3	Greenwich date (year)	2003 =LctGYear(C3,C4,C5,C6,C7,C8,C9,C10)								
20	4	planet ecl long (deg)	166.1186259 =PlanetLong(C3,C4,C5,C6,C7,C8,C9,C10,C11)								
21	5	planet ecl lat (deg)	1.035218287 =PlanetLat(C3,C4,C5,C6,C7,C8,C9,C10,C11)								
22	6	planet RA (hours)	11.17527609 =DDDH(ECRA(C16,0,0,C17,0,0,C8,C9,C10))								
23	7	planet dec (deg)	6.430406407 =ECDec(C16,0,0,C17,0,0,C8,C9,C10)								

Figure 69. Finding the position of a planet by a more precise method.

57 The distance, light-travel time and angular size of a planet

During the course of our calculations in Section 54 to determine the position of a planet we found the distances r and R of the planet and the Earth, respectively, from the Sun. We can quite easily use these values together with the heliocentric longitudes l and L to calculate the planet's distance, ρ , from the Earth. The situation is drawn in Figure 70. The planet, P, does not lie in the plane of the ecliptic, so its heliocentric latitude, ψ , must be taken into account. The formula is

$$\rho^2 = R^2 + r^2 - 2Rr \cos(l - L) \cos \psi.$$

Let $\psi = 0$ for approximations

It is usual to express r and R in **astronomical units** (AU) where 1 AU is the semi-major axis of the Earth's orbit. ρ is then the distance of the planet from the Earth measured in AU.

Having calculated ρ , it is then an easy matter to find the light-travel time, τ , the time taken for the light to reach us from the planet. When we view a planet now, we see it in the position it occupied τ hours ago, given by

$$\tau = 0.1386\rho \text{ hours,}$$

where ρ is expressed in AU.

We can also find the apparent angular diameter, θ , of the planet given by

$$\theta = \frac{\theta_0}{\rho},$$

where ρ is again expressed in AU and θ_0 is the angular diameter of the planet when it is at 1 AU from the Earth. Values of θ_0 are given in Table 8.

We shall calculate the distance, the light-travel time and the apparent angular diameter of Jupiter on 22 November 2003.

Method	Example
1. Find r , R , l_p , L and ψ as in §54.	$r = 5.397\,121 \text{ AU}$ $R = 0.987\,847 \text{ AU}$ $l_p = 156.236\,900 \text{ degrees}$ $L = 59.274\,748 \text{ degrees}$ $\psi = 1.076\,044 \text{ degrees}$
2. Calculate $\rho^2 = R^2 + r^2 - 2Rr \cos(l_p - L) \cos \psi$.	$\rho^2 = 31.397\,037 \text{ AU}^2$
3. Take the square root to find ρ .	$\rho = 5.603 \text{ AU}$
4. Multiply by 0.1386 to find τ ; convert to minutes and seconds (§8).	$\tau = 46\text{m } 36\text{s}$
5. Find θ from $\theta = \frac{\theta_0}{\rho}$.	$\theta = 35.1 \text{ arcsec}$

The *Astronomical Almanac* quotes $\rho = 5.60 \text{ AU}$, $\tau = 46\text{m } 34\text{s}$ and $\theta = 35.0''$ for Jupiter on this day.

The spreadsheet for this calculation is called PlanetVis and includes other calculations to do with the visual aspect of a planet. It is given in Section 60, Figure 72. The corresponding spreadsheet functions are defined there too.

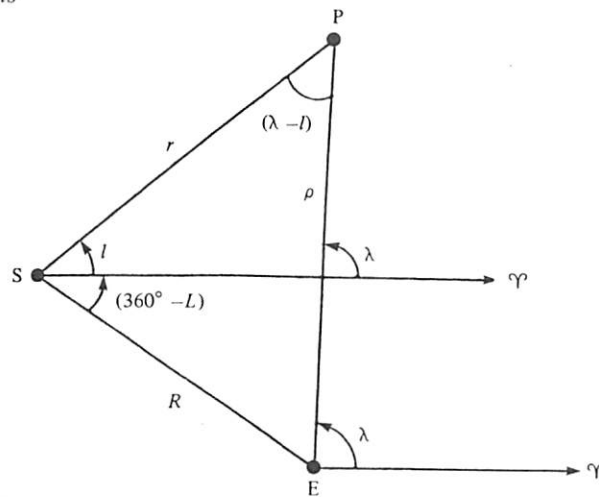


Figure 70. Finding the distance of a planet.

58 The phases of the planets

At any point in the orbit of a planet, the hemisphere which faces towards the Sun is brightly illuminated while the other half of the planet's surface is dark. The fraction of the surface that we can see from the Earth, however, is that part lying on the hemisphere facing the Earth which usually overlaps both the bright and the dark sides. We are presented therefore with a view of the planet's disc that is not uniformly illuminated but which contains a bright segment, the rest of the disc being dark and usually invisible. As the relative positions of the Earth, the planet and the Sun vary, so the area of the visible disc that is illuminated changes. The **phase** is defined to be the fraction of the visible disc that is illuminated.

In Figure 70, the angle $(\lambda - l)$ at P is the solar elongation of the Earth as measured at the planet. We represent this angle by d . Thus

$$d = \lambda - l.$$

The phase, F , is related to d by the formula

$$F = \frac{1}{2}(1 + \cos d).$$

F always lies in the range 0 to 1. When $F = 0$, the whole of the dark side of the planet is towards the Earth. This can only happen for the inner planets Mercury and Venus. When $F = 1$, the whole of the bright side faces the Earth.

We shall find the phases of Mercury and Jupiter on 22 November 2003 as our example.

Method	Example
1. Calculate $d = \lambda - l$ using the method outlined in §54 to find λ and l .	Mercury: $d_1 = -34.082^\circ$ Jupiter: $d_2 = 10.736^\circ$
2. Find $F = \frac{1}{2}(1 + \cos d)$.	Mercury: $F_1 = 0.91$ Jupiter: $F_2 = 0.99$

The *Astronomical Almanac* gives phase values of 0.90 and 0.99 for Mercury and Jupiter respectively.

The spreadsheet for this calculation is called PlanetVis and includes other calculations to do with the visual aspect of a planet. It is given in Section 60, Figure 72. The corresponding spreadsheet functions are defined there too.

59 The position-angle of the bright limb

Figure 71 shows the appearance of a planet whose phase is about $F = 0.7$. The dashed outline is of that part of the disc which is invisible, and the line NS is the projection of the Earth's axis onto the disc. The terminator, the line dividing night from day, is the curve AB. Position-angles are measured anticlockwise from the north. Thus points A and B are at position-angles θ_1 and θ_2 . The point C, halfway between A and B on the circumference of the disc, is the midpoint of the bright side and it defines the position-angle, χ , of the bright limb. Hence

$$\chi = \frac{1}{2}(\theta_1 + \theta_2).$$

We can easily calculate χ provided we know the equatorial coordinates of the planet (α, δ) and of the Sun $(\alpha_\odot, \delta_\odot)$. Then

$$\chi = \tan^{-1} \left\{ \frac{\cos \delta_\odot \sin(\alpha_\odot - \alpha)}{\cos \delta \sin \delta_\odot - \sin \delta \cos \delta_\odot \cos(\alpha_\odot - \alpha)} \right\}.$$

For example, what was the position-angle of the bright limb of Mercury on 22 November 2003? The Sun's coordinates were $\alpha_\odot = 15^{\text{h}} 48^{\text{m}} 13^{\text{s}}$, $\delta_\odot = -19^\circ 59' 32''$, and Mercury's coordinates were $\alpha = 16^{\text{h}} 52^{\text{m}} 02^{\text{s}}$, $\delta = -24^\circ 38' 41''$.

Method	Example
1. Find the right ascension and declination of the planet (§54).	$\alpha = 16^{\text{h}} 52^{\text{m}} 02^{\text{s}}$ $\delta = -24^{\circ} 38' 41''$
2. Find the coordinates of the Sun (§46).	$\alpha_{\odot} = 15^{\text{h}} 48^{\text{m}} 13^{\text{s}}$ $\delta_{\odot} = -19^{\circ} 59' 32''$
3. Convert both sets of coordinates to decimal form (§§7 and 21).	$\alpha = 16.867222$ hours $\delta = -24.644722$ degrees $\alpha_{\odot} = 15.803611$ hours $\delta_{\odot} = -19.992222$ degrees
4. Find $\Delta\alpha = \alpha_{\odot} - \alpha$. Convert to degrees by multiplying by 15 (§22).	$\Delta\alpha = -15.954165$ degrees
5. Find $y = \cos \delta_{\odot} \sin \Delta\alpha$.	$y = -0.258304$
6. Find $x = \cos \delta \sin \delta_{\odot} - \sin \delta \cos \delta_{\odot} \cos \Delta\alpha$.	$x = 0.066018$
7. Find $\chi = \tan^{-1} \left(\frac{y}{x} \right)$. Remove the ambiguity from taking inverse tan using the signs of x and y , referring to Figure 29, and adding or subtracting 180 if necessary to bring the result into the correct quadrant.	$\chi = -75.663043$ degrees (already in correct quadrant; add 360 to bring into the range 0–360) $\chi = 284.3$ degrees

The spreadsheet for this calculation is called PlanetVis and includes other calculations to do with the visual aspect of a planet. It is given in Section 60, Figure 72. The corresponding spreadsheet functions are defined there too.

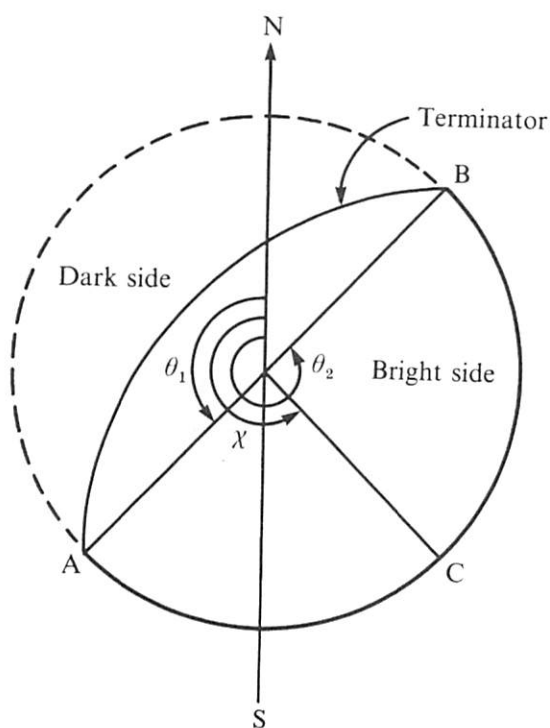


Figure 71. The position-angle of the bright limb.

60 The apparent brightness of a planet

Our calculations so far have given us the position, the solar elongation (Section 52), the distance from the Earth, the apparent angular diameter, the phase, and the position-angle of the bright limb of a planet. We need only add the apparent brightness to the list to obtain all the important parameters of the planet's **visual aspect**.

Brightness is usually measured in **magnitudes**, m , on a non-linear scale such that decreasing brightness goes with increasing magnitude. The brightest stars have a magnitude of about 1 while the faintest stars just visible with the unaided eye are of magnitude 6. The ratio in the light power flux between one magnitude and the next is about 2.5. The Sun, very bright at the Earth, has a visual magnitude of -26.74 while the Moon's magnitude at opposition is -12.73 . The planets range from about $m = -4$ for Venus at its most brilliant to $m = +14$ for Pluto (strictly no longer classified as a planet) at its brightest.

The variation in a planet's brightness is caused by several factors. First the Sun's light flux on the planet varies inversely as the square of its distance, r , from the Sun. Then the amount of that light reradiated towards the Earth depends on the phase, F , and a 'brightness factor', V_0 , the latter being a measure of the reflectivity of the planet combined with the area of the planet's disc. The larger the planet's area, the more light it intercepts from the Sun and hence the more it radiates towards the Earth. Finally, the light flux received from the planet varies inversely as the square of the planet's distance, ρ , from the Earth.

We can obtain an approximate value for the apparent magnitude of a planet from the formula

$$m = 5 \log_{10} \left(\frac{r\rho}{\sqrt{F}} \right) + V_0,$$

where r and ρ are measured in AU. The values of V_0 are listed in Table 8.

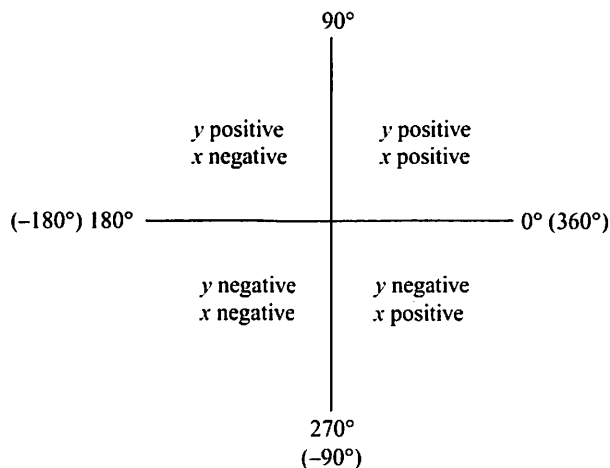
As an example, let us calculate the apparent magnitude of Jupiter on 22 November 2003.

Method	Example
1. Find the values of r , ρ and F using the methods given in §§54, 57 and 58.	$r = 5.397$ AU $\rho = 5.603$ AU $F = 0.99$
2. Calculate $m = 5 \log_{10} \left(\frac{r\rho}{\sqrt{F}} \right) + V_0$. r and ρ must both be expressed in AU.	$m = -2$

The value of m given in the *Astronomical Almanac* for Jupiter on 22 November 2003 is $m = -1.9$. In general, our calculations should be correct to within a magnitude or so. No account has been taken of **atmospheric extinction** (see Section 43), which can increase the apparent magnitude of a star or planet near the horizon by 2 or 3. Nonetheless, our calculations will provide a fair guide of what to expect.

The distance, light time, angular diameter, phase, position-angle of the bright limb, and the approximate magnitude of a planet are all parts of its visual aspect (i.e. its appearance when viewed from Earth) and these calculations are swept up into one spreadsheet called PlanetVis, Figure 72. As in PlanetPos1 (Section 54), we make use of the spreadsheet function VLOOKUP to extract data from a table contained on a second spreadsheet (rows 22 and 31). We have also defined three additional spreadsheet functions to make life easier. The first of these to appear in the spreadsheet (row 20) is PlanetDist, taking nine arguments which are the local civil time expressed as hours, minutes, and seconds, the daylight saving and time zone offsets in hours, the local calendar date expressed as day, month, year, and the planet full name as a string

(upper or lower case, or a combination). This function returns the Earth–planet distance in AU. The next new spreadsheet function to appear is called `PlanetHLong1` (row 23) which returns the planet's heliocentric orbital longitude in degrees. It takes the same nine arguments as `PlanetDist`. The third new spreadsheet function is called `PlanetRVect` (row 30) and, as its name suggests, it returns the distance of the planet from the Sun (i.e. the length of its radius vector) in AU; it has the same nine arguments as `PlanetDist`.

Figure 29. Removing the ambiguity on taking $\tan^{-1}(y/x)$.

Ecliptic to Equatorial Coordinate Conversion									
3	Input	ecl long deg	139	Output	RA hour	9 = DHHour(C27)			
4		ecl long min	41		RA min	34 = DHMin(C27)			
5		ecl long sec	10		RA sec	53.4 = DHSec(C27)			
6		ecl lat deg	4		dec deg	19 = DDDeg(C21)			
7		ecl lat min	52		dec min	32 = DDMin(C21)			
8		ecl lat sec	31		dec sec	8.52 = DDSec(C21)			
9		G day	6						
10		G month	7						
11		G year	2009						
13	1	eclon (deg)	139.6861111 =DMSDD(C3,C4,C5)						
14	2	eclat (deg)	4.875277778 =DMSDD(C6,C7,C8)						
15	3	eclon (rad)	2.437982558 =RADIANS(C13)						
16	4	eclat (rad)	0.085089649 =RADIANS(C14)						
17	5	obliq (deg)	23.43923176 =Obliq(C9,C10,C11)						
18	6	obliq (rad)	0.409091768 =RADIANS(C17)						
19	7	sin dec	0.334394125 =SIN(C16)*COS(C18)+COS(C16)*SIN(C18)*SIN(C15)						
20	8	dec (rad)	0.340962273 =ASIN(C19)						
21	9	dec (deg)	19.53569924 =DEGREES(C20)						
22	10	y	0.559659329 =SIN(C15)*COS(C18)-TAN(C16)*SIN(C18)						
23	11	x	-0.762511521 =COS(C15)						
24	12	RA (rad)	2.508430994 =ATAN2(C23,C22)						
25	13	RA (deg)	143.7225092 =DEGREES(C24)						
26	14	RA (deg)	143.7225092 =C25-360*INT(C25/360)						
27	15	RA (hours)	9.58150061 =DDDH(C26)						

Figure 30. Converting from ecliptic to equatorial coordinates.