PHY/EGR 321.001

Spring 2009

Useful Equations

Chapter 11 – Kinematics of Particles

Rectilinear Motion

$$a = \frac{dv}{dt} = v\frac{dv}{dx}$$

Curvilinear Motion

$$\vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a} = \frac{d\vec{v}}{dt}$$

Motion Relative to a Frame in Translation

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A} \qquad \vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \qquad \vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

Tangential and Normal Components

$$\vec{v} = v\hat{e}_t \qquad \vec{a} = \frac{dv}{dt}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

Radial and Transverse Components

$$\vec{r} = r\hat{e}_r \qquad \vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

Chapter 12 – Kinetics of Particles: Newton's Second Law

Newton's Second Law

$$\sum \vec{F} = m\vec{a} = \dot{\vec{L}} \qquad \vec{H}_o = \vec{r} \times m\vec{v}$$

$$\vec{L} = m\vec{v} \qquad \sum \vec{M}_o = \dot{\vec{H}}_o$$

Central Force

$$\dot{\vec{H}}_o = 0 \qquad h = r^2\dot{\theta}$$

Universal Law of Gravitation

$$F = G\frac{Mm}{r^2}$$

Conic Sections

$$\frac{1}{r} = \frac{GM}{h^2} + C\cos\theta = \frac{GM}{h^2}(1 + \varepsilon\cos\theta)$$

Period for Elliptical Orbit

$$\tau = \frac{2\pi ab}{h} \quad \text{where } a = \frac{1}{2}(r_0 + r_1) \quad \text{and} \quad b = \sqrt{r_o r_1}$$

For Circular Orbit

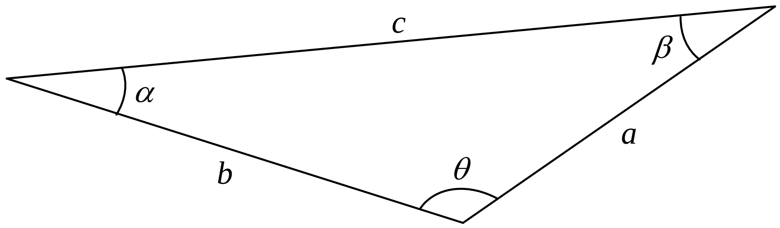
$$v = \sqrt{\frac{GM}{r}}$$

For Escape Velocity

$$v = \sqrt{\frac{2GM}{r}}$$

Law of Sines

$$\frac{a}{sin\,\alpha} = \frac{b}{sin\,\beta} = \frac{c}{sin\,\theta}$$



Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

Chapter 13 – Kinetics of Particles: Energy and Momentum Methods

Work of a Force

$$U_{1\to 2} = \int_1^2 \vec{F} \cdot d\vec{r}$$

Work of the Force of Gravity

$$U_{1\to 2} = -W\Delta y$$

Work of the Force Exerted by a Spring

$$U_{1\to 2} = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2$$

Work of a Gravitational Force

$$U_{1\to 2} = \frac{GMm}{r_2} - \frac{GMm}{r_1}$$

Kinetic Energy

$$T = \tfrac{1}{2}mv^2$$

Principle of Work and Energy

$$T_1 + U_{1\to 2} = T_2$$

Power

$$Power = \frac{dU}{dt} = \vec{F} \cdot \vec{v}$$

Gravitational PE (Close to Earth Surface)

$$V_g = Wy$$

GPE (Not So Close to Earth Surface)

$$V_g = -\frac{GMm}{r}$$

Elastic Potential Energy

$$V_e = \tfrac{1}{2}kx^2$$

Conservative Forces

$$U_{1\to 2} = V_1 - V_2$$

$$\oint \vec{F} \cdot d\vec{r} = 0$$

$$dU = -dV(x, y, z)$$

$$\vec{F} = -\vec{\nabla} V$$

Conservation of Energy

$$T_1 + V_1 = T_2 + V_2$$

Principle of Impulse and Momentum

$$\boldsymbol{Imp}_{1\to 2} = \int_{t_1}^{t_2} \vec{F} dt$$

$$m\vec{v}_1 + \boldsymbol{Imp}_{1\to 2} = m\vec{v}_2$$

Direct Central Impact

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$v_B' - v_A' = e(v_A - v_B)$$

Chapter 14 – Systems of Particles

System of n Particles

$$\sum_{i=1}^{n}\vec{F}_i=\sum_{i=1}^{n}m_i\vec{a}_i$$

$$\sum_{i=1}^{n}(\vec{r}_i x\vec{F}_i)=\sum_{i=1}^{n}(\vec{r}_i x m_i\vec{a}_i)$$

Linear and Angular Momentum of a System of n Particles

$$\vec{L}=\sum_{i=1}^{n}m_i\vec{v}_i$$

$$\vec{H}_o=\sum_{i=1}^{n}(\vec{r}_i x m_i\vec{v}_i)$$

$$\sum_{i=1}^{n}\vec{F}_i=\dot{\vec{L}}$$

$$\sum_{i=1}^{n}(\vec{M}_o)_i=\dot{\vec{H}}_o$$

Center of Mass

$$m\vec{\bar{r}}=\sum_{i=1}^{n}m_i\vec{r}_i$$

$$\vec{L}=m\vec{\bar{v}}$$

$$\dot{\vec{L}}=m\vec{\bar{a}}$$

$$\sum_{i=1}^{n}\vec{F}_i=\dot{\vec{L}}=m\vec{\bar{a}}$$

Angular Momentum of a System of n Particles

$$\sum_{i=1}^{n}(\vec{M}_G)_i=\dot{\vec{H}}'_G=\dot{\vec{H}}_G$$

$$\vec{H}_G=\sum_{i=1}^{n}(\vec{r}_i' x m_i\vec{v}_i)=\sum_{i=1}^{n}(\vec{r}_i' x m_i\vec{v}_i')$$

Conservation of Momentum

$$\vec{L}=constant \qquad and \qquad \vec{H}_o=constant$$

Kinetic Energy of a System of n Particles

$$T=\tfrac{1}{2}\sum_{i=1}^{n}m_i v_i^2$$

$$T=\tfrac{1}{2}m\bar{v}^2+\tfrac{1}{2}\sum_{i=1}^{n}m_i v_i'^2$$

Work-Energy Principle for a System of n Particles

$$T_1+U_{1\to 2}=T_2$$

Principle of Impulse and Momentum for a System of n Particles

$$\vec{L}_1+\sum_{i=1}^{n}\int_{t_1}^{t_2}\vec{F}_i\,dt=\vec{L}_2$$

$$(\vec{H}_o)_1+\sum_{i=1}^{n}\int_{t_1}^{t_2}(\vec{M}_o)_i\,dt=(\vec{H}_o)_2$$

Steady Stream of Particles

$$\sum_{i=1}^{n}\vec{F}_i=\frac{dm}{dt}(\vec{v}_B-\vec{v}_A)$$

Systems Gaining or Losing Mass

$$\sum_{i=1}^{n}\vec{F}_i=m\frac{d\vec{v}}{dt}-\frac{dm}{dt}\vec{u}$$

Chapter 15 – Kinematics of Rigid Bodies

Translation

$$\vec{v}_B = \vec{v}_A \text{ and } \vec{a}_B = \vec{a}_A$$

Rotation about a Fixed Axis and a Fixed Point

$$\vec{v} = \vec{\omega} \times \vec{r} \text{ and } \vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

Absolute and Relative Velocity and Acceleration in Plane Motion

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

Rate of change of a Vector with Respect to a Rotating Frame

$$(\dot{\vec{Q}})_{OXYZ} = (\dot{\vec{Q}})_{Oxyz} + \vec{\Omega} \times \vec{Q}$$

General Motion

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$
$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A})$$

Plane Motion of a Particle Relative to a Rotating Frame. Coriolis Acceleration

$$\vec{v}_P = (\dot{\vec{r}})_{OXY} = \vec{\Omega} \times \vec{r} + (\dot{\vec{r}})_{Oxy}$$
$$\vec{v}_P = \vec{v}_{P'} + \vec{v}_{P/F}$$
$$\vec{a}_P = \dot{\vec{\Omega}} \times \vec{r} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) + 2\vec{\Omega} \times (\dot{\vec{r}})_{Oxy} + (\ddot{\vec{r}})_{Oxy}$$
$$\vec{a}_P = \vec{a}_{P'} + \vec{a}_{P/F} + \vec{a}_c$$

Three-Dimensional Motion of a Particle Relative to a Rotating Frame. Coriolis Acceleration

$$\vec{v}_P = (\dot{\vec{r}})_{OXYZ} = \vec{\Omega} \times \vec{r} + (\dot{\vec{r}})_{Oxyz}$$
$$\vec{v}_P = \vec{v}_{P'} + \vec{v}_{P/F}$$
$$\vec{a}_P = \dot{\vec{\Omega}} \times \vec{r} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) + 2\vec{\Omega} \times (\dot{\vec{r}})_{Oxyz} + (\ddot{\vec{r}})_{Oxyz}$$
$$\vec{a}_P = \vec{a}_{P'} + \vec{a}_{P/F} + \vec{a}_c$$

Frame of Reference in General Motion

$$\vec{v}_P = (\dot{\vec{r}})_{OXYZ} = \vec{v}_A + \vec{\Omega} \times \vec{r}_{P/A} + (\dot{\vec{r}}_{P/A})_{Axyz}$$
$$\vec{v}_P = \vec{v}_{P'} + \vec{v}_{P/F}$$
$$\vec{a}_P = \vec{a}_A + \dot{\vec{\Omega}} \times \vec{r}_{P/A} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{P/A})$$
$$+ 2\vec{\Omega} \times (\dot{\vec{r}}_{P/A})_{Axyz} + (\ddot{\vec{r}}_{P/A})_{Axyz}$$

$$\vec{a}_P = \vec{a}_{P'} + \vec{a}_{P/F} + \vec{a}_c$$

Chapter 16 – Plane Motion of Rigid Bodies: Forces and Accelerations

Equations of Motion for a Rigid Body

$$\sum \vec{F} = m\bar{\vec{a}} \text{ and } \sum \vec{M}_G = \dot{\vec{H}}_G$$

Angular Momentum of a Rigid Body in Plane Motion

$$\vec{H}_G = \bar{I}\vec{\omega}$$
$$\dot{\vec{H}}_G = \bar{I}\dot{\vec{\omega}} = \bar{I}\vec{\alpha}$$

Noncentroidal Rotation

$$\bar{a}_t = \bar{r}\alpha \text{ and } \bar{a}_n = \bar{r}\omega^2$$
$$\sum M_O = I_O\alpha$$

Rolling Motion

$$\bar{a} = r\alpha$$

Chapter 17 – Plane Motion of Rigid Bodies: Energy and Momentum Methods

Principle of Work and Energy for a Rigid Body

$$T_1 + U_{1\to 2} = T_2$$

Work of Forces Acting on a Rigid Body

$$U_{1\to 2} = \int_{A_1}^{A_2} \vec{F} \cdot d\vec{r}$$

$$U_{1\to 2} = \int_{\theta_1}^{\theta_2} M d\theta$$

Kinetic Energy of a Rigid Body in Plane Motion

$$T = \tfrac{1}{2} m\bar{v}^2 + \tfrac{1}{2} \bar{I}\omega^2$$

Noncentroidal Rotation

$$T = \tfrac{1}{2} I_o \omega^2$$

Power

$$P = \frac{dU}{dt} = \vec{F} \cdot \vec{v}$$

$$P = \frac{dU}{dt} = \frac{Md\theta}{dt} = M\omega$$

Principle of Impulse and Momentum for the Plane Motion of a Rigid Body

$$\textbf{Syst Momentum}_{\mathbf{1}} + \textbf{Syst Ext Imp}_{\mathbf{1\to 2}} = \textbf{Syst Momenta}_{\mathbf{2}}$$

Noncentroidal Rotation

$$\bar{I}\omega + (m\bar{r}\omega)\bar{r} = (\bar{I} + m\bar{r}^2)\omega = I_o\omega$$

$$I_o\omega_1 + \sum \int_{t_1}^{t_2} M_o dt = I_o\omega_2$$

Eccentric Impact

$$(v'_B)_n - (v'_A)_n = e[(v_A)_n - (v_B)_n]$$

Chapter 18 – Kinetics of Rigid Bodies in Three Dimensions

Fundamental Equations for Rigid Body in Three Dimensions

$$\sum \vec{F} = m\bar{\vec{a}} \text{ and } \sum \vec{M}_G = \dot{\vec{H}}_G$$

Moments of Inertia and Products of Inertia

$$\bar{I}_x = \int (y^2 + z^2)dm, \quad \bar{I}_y = \int (x^2 + z^2)dm, \ \bar{I}_z = \int (x^2 + y^2)dm$$

$$\bar{I}_{xy} = \int xy\ dm, \ \bar{I}_{xz} = \int xz\ dm, \ \bar{I}_{yz} = \int yz\ dm$$

Angular Momentum of a Body about its Mass Center

$$\begin{aligned} H_x &= +\bar{I}_x\omega_x - \bar{I}_{xy}\omega_y - \bar{I}_{xz}\omega_z \\ H_y &= -\bar{I}_{yx}\omega_x + \bar{I}_y\omega_y - \bar{I}_{yz}\omega_z \\ H_z &= -\bar{I}_{zx}\omega_x - \bar{I}_{zy}\omega_y + \bar{I}_z\omega_z \end{aligned}$$

Angular Momentum about a given point O

$$\vec{H}_O = \bar{\vec{r}}\ x\ m\bar{\vec{v}} + \vec{H}_G$$

Angular Momentum of a Body about a Fixed Point

$$\begin{aligned} H_x &= +I_x\omega_x - I_{xy}\omega_y - I_{xz}\omega_z \\ H_y &= -I_{yx}\omega_x + I_y\omega_y - I_{yz}\omega_z \\ H_z &= -I_{zx}\omega_x - I_{zy}\omega_y + I_z\omega_z \end{aligned}$$

Kinetic Energy of a Rigid Body in Three Dimensions

$$T = \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}(\bar{I}_x\omega_x^2 + \bar{I}_y\omega_y^2 + \bar{I}_z\omega_z^2 - 2\bar{I}_{xy}\omega_x\omega_y - 2\bar{I}_{yz}\omega_y\omega_z - 2\bar{I}_{zx}\omega_z\omega_x)$$

Kinetic Energy of a Rigid Body about a Fixed Point

$$T = \tfrac{1}{2}(I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2 - 2I_{xy}\omega_x\omega_y - 2I_{yz}\omega_y\omega_z - 2I_{zx}\omega_z\omega_x)$$

Relationship between H_G in Rotating Frame to H_G in Fixed Frame

$$\dot{\vec{\mathbf{H}}}_{\mathbf{G}} = (\dot{\vec{\mathbf{H}}}_{\mathbf{G}})_{\mathbf{Gxyz}} + \vec{\Omega}\ \mathbf{x}\ \vec{\mathbf{H}}_{\mathbf{G}} = \sum \vec{\mathbf{M}}_{\mathbf{G}}$$

Euler's Equations

$$\sum M_x = \bar{I}_x \dot{\omega}_x - (\bar{I}_y - \bar{I}_z)\omega_y \omega_z$$
$$\sum M_y = \bar{I}_y \dot{\omega}_y - (\bar{I}_z - \bar{I}_x)\omega_z \omega_x$$
$$\sum M_z = \bar{I}_z \dot{\omega}_z - (\bar{I}_x - \bar{I}_y)\omega_x \omega_y$$

Motion of a Rigid body about a Fixed Point

$$\dot{\vec{H}}_O = (\dot{\vec{H}}_O)_{Oxyz} + \vec{\Omega}\ x\ \vec{H}_O = \sum \vec{M}_O$$

Rotation of a Rigid Body About a Fixed Axis (z)

$$\sum M_x = -I_{xz}\alpha + I_{yz}\omega^2$$
$$\sum M_y = -I_{yz}\alpha - I_{xz}\omega^2$$
$$\sum M_z = I_z \alpha$$